Convergence to equilibrium under a random Hamiltonian

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We analyse the time of equilibration of subsystems of a larger system under a random total Hamiltonian, in which the basis of the Hamiltonian is drawn from the Haar measure. We obtain that the time of equilibration is of the order of the inverse of the arithmetic average of the Bohr frequencies. To compute the average over random basis, we compute the inverse of a matrix of overlaps of operators which permute four systems. We first obtain results on such matrix for a representation of an arbitrary finite group and then apply it to the particular representation of the permutation group under consideration.

A system equilibrates when diagonalizing its hamiltonian is sufficiently hard

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Understanding the mechanisms responsible for the equilibration of isolated quantum many-body systems is a longstanding open problem. Some of them have been identified, but a complete picture is still missing. In this work we obtain a link between equilibration and the computational complexity of diagonalizing the hamiltonian, quantified by the circuit-complexity of the unitary that maps the local basis to the energy eigenbasis. We show that, almost all hamiltonians whose diagonalizing unitary has complexity at least quadratic in the system's size, enjoy local equilibration. Also, for a relevant class of systems, almost all hamiltonians with less than linear complexity do not equilibrate. We show that the dynamics of the convergence to equilibrium, and its time scale, are given by the Fourier transform of the level density, and calculate this for two cases. In the limit of large complexity, we recover the equilibration properties of hamiltonians given by random matrix theory. We argue that these results also hold for other measures of complexity.

Subsystem's dynamics under random Hamiltonian evolution

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We study time evolution of a subsystem's density matrix under a unitary evolution, generated by a sufficiently complex, say quantum chaotic, Hamiltonian. We exactly calculate all coherences, purity and fluctuations. The reduced density matrix is described in terms of a noncentral correlated Wishart ensemble. Our description accounts for a transition from an arbitrary initial state towards a random state at large times, enabling us to determine the convergence time after which random states are reached. We identify and describe a number of other interesting features, like a series of collisions between the largest eigenvalue and the bulk, accompanied by a phase transition in its distribution function.

Violation of a Temporal Bell Inequality for Single Spins in a Diamond Defect Center

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Quantum nonlocality has been experimentally investigated by testing different forms of Bell's inequality, yet a loophole-free realization has not been achieved up to now. Much less explored are temporal Bell inequalities, which are not subject to the locality assumption, but impose a constraint on the system's time correlations. In this Letter, we report on the experimental violation of a temporal Bell's inequality using a nitrogen-vacancy (NV) defect in diamond and provide a novel quantitative test of quantum coherence. Such a test requires strong control over the system, and we present a new technique to initialize the electronic state of the NV with high fidelity, a necessary requirement also for reliable quantum information processing and/or the implementation of protocols for quantum metrology.

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Conditions for Monogamy of Quantum Discord: Monogamous Greenberger-Horne-Zeilinger versus Polygamous W states

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In the quantum world, notions like entanglement and quantum discord, are expected to respect all the conditions required for them to be good measures of quantum correlations in the bipartite scenario. In a multipartite setting, sharing entanglement between several parties is restricted by the monogamy of entanglement. We take over the concept of monogamy to quantum discord and find that it does not satisfy monogamy in general. We identify necessary and sufficient conditions for quantum discord to obey monogamy. As an application, we show that while three-qubit generalized Greenberger-Horne-Zeilinger states follow monogamy, generalized W states do not.

Quantum thermal equilibration from microcanonical equipartition

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The problem of mutual equilibration between two finite quantum systems, A and B, prepared initially at different temperatures is elucidated. The energy exchange between the two systems is mediated by a weak interaction. We predict that the sole assumption of microcanonical equipartition inside the energy shells of the total composite system, $A \otimes B$ is sufficient to obtain a relaxation of the halves, A and B, towards common thermal-like states at the same 'equilibrium' temperature. This conjecture is fully corroborated by an exact diagonalization procedure for three different classes of quantum Hamiltonians.

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Topological Geometric Entanglement of Blocks

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Here we establish the connection between topological order and the geometric entanglement, as measured by the logarithm of the overlap between a given state and its closest product state of blocks. As happens for the entanglement entropy, we find that the geometric entanglement is the sum of two terms: a non-universal one obeying a boundary law times the number of blocks, and a universal one quantifying the underlying long-range entanglement of a topologically-ordered state. For simplicity we focus on the case of Kitaev's toric code model.

Alternatives to Eigenstate Thermalization

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An isolated quantum many-body system in an initial pure state will come to thermal equilibrium if it satisfies the *eigenstate thermalization hypothesis* (ETH). We consider alternatives to ETH that have been proposed. We first show that von Neumann's *quantum ergodic theorem* (QET) relies on an assumption that is essentially equivalent to ETH. We also investigate whether initial states produced by a quench can lead to thermal behavior in systems that do not obey ETH after the quench, namely, integrable systems. We find examples of this, but only for initial states that obeyed ETH before the quench.

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Simulating typical entanglement with many-body Hamiltonian dynamics

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We study the time evolution of the amount of entanglement generated by one dimensional spin-1/2 Ising-type Hamiltonians composed of many-body interactions. We investigate sets of states randomly selected during the time evolution generated by several types of time-independent Hamiltonians by analyzing the distributions of the amount of entanglement of the sets. We compare such entanglement distributions with that of typical entanglement, entanglement of a set of states randomly selected from a Hilbert space with respect to the unitarily invariant measure. We show that the entanglement distribution obtained by a time-independent Hamiltonian can simulate the average and standard deviation of the typical entanglement, if the Hamiltonian contains suitable many-body interactions. We also show that the time required to achieve such a distribution is polynomial in the system size for certain types of Hamiltonians.

Local Realism of Macroscopic Correlations

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We identify conditions under which correlations resulting from quantum measurements performed on macroscopic systems (systems composed of a number of particles of the order of the Avogadro number) can be described by local realism. We argue that the emergence of local realism at the macroscopic level is caused by an interplay between the monogamous nature of quantum correlations and the fact that macroscopic measurements do not reveal properties of individual particles.

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Non-Markovian effects on non-locality of a Schrödinger cat state

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Non-Markovian evolutions are responsible for a wide variety of physically interesting effects. Here, we study non-locality of the non-classical state of a system consisting of a qubit and an oscillator exposed to the effects of non-Markovian evolutions. We find that the different facets of non-Markovianity affect non-locality in different and non-obvious ways: ranging from pronounced insensitivity of the Bell function to quite a spectacular evidence of information kick-back.

Specker's Parable of the Over-protective Seer: A Road to Contextuality, Nonlocality and Complementarity

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In 1960, the mathematician Ernst Specker described a simple example of nonclassical correlations, the counterintuitive features of which he dramatized using a parable about a seer who sets an impossible prediction task to his daughter's suitors. We revisit this example here, using it as an entrée to three central concepts in quantum foundations: contextuality, Bell-nonlocality, and complementarity. Specifically, we show that Specker's parable offers a narrative thread that weaves together a large number of results, including: the impossibility of measurement-noncontextual and outcome-deterministic ontological models of quantum theory (the 1967 Kochen-Specker theorem), in particular the recent state-specific pentagram proof of Klyachko; the impossibility of Bell-local models of quantum theory (Bell's theorem), especially the proofs by Mermin and Hardy and extensions thereof; the impossibility of a preparation-noncontextual ontological model of quantum theory; and the existence of triples of positive operator valued measures (POVMs) that can be measured jointly pairwise but not triplewise. Along the way, several novel results are presented, including: a generalization of a theorem by Fine connecting the existence of a joint distribution over outcomes of counterfactual measurements to the existence of a measurement-noncontextual and outcome-deterministic ontological model; a generalization of Klyachko's proof of the Kochen-Specker theorem from pentagrams to a family of star polygons; a proof of the Kochen-Specker theorem in the style of Hardy's proof of Bell's theorem (i.e., one that makes use of the failure of the transitivity of implication for counterfactual statements); a categorization of contextual and Bell-nonlocal correlations in terms of frustrated networks; a derivation of a new inequality testing preparation noncontextuality; and lastly, some novel results on the joint measurability of POVMs and the question of whether these can be modeled noncontextually. Finally, we emphasize that Specker's parable of the over-protective seer provides a novel type of foil to quantum theory, challenging us to explain why the particular sort of contextuality and complementarity embodied therein does not arise in a quantum world.

Why Philosophers Should Care About Computational Complexity

Scott Aaronson*

Abstract

One might think that, once we know something is computable, how *efficiently* it can be computed is a practical question with little further philosophical importance. In this essay, I offer a detailed case that one would be wrong. In particular, I argue that *computational complexity theory*—the field that studies the resources (such as time, space, and randomness) needed to solve computational problems—leads to new perspectives on the nature of mathematical knowledge, the strong AI debate, computationalism, the problem of logical omniscience, Hume's problem of induction, Goodman's grue riddle, the foundations of quantum mechanics, economic rationality, closed timelike curves, and several other topics of philosophical interest. I end by discussing aspects of complexity theory itself that could benefit from philosophical analysis.

LETTER

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Microwave quantum logic gates for trapped ions

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Control over physical systems at the quantum level is important in fields as diverse as metrology, information processing, simulation and chemistry. For trapped atomic ions, the quantized motional and internal degrees of freedom can be coherently manipulated with laser light[1,2]. Similar control is difficult to achieve with radio-frequency or microwave radiation: the essential coupling between internal degrees of freedom and motion requires significant field changes over the extent of the atoms' motion[2,3], but such changes are negligible at these frequencies for freely propagating fields. An exception is in the near field of microwave currents in structures smaller than the free-space wavelength[4,5], where stronger gradients can be generated. Here we first manipulate coherently (on time-scales of 20 nanoseconds) the internal quantum states of ions held in a microfabricated trap. The controlling magnetic fields are generated by microwave currents in electrodes that are integrated into the trap structure. We also generate entanglement between the internal degrees of freedom of two atoms with a gate operation[4,6–8] suitable for general quantum computation[9]; the entangled state has a fidelity of 0.76(3), where the uncertainty denotes standard error of the mean. Our approach, which involves integrating the quantum control mechanism into the trapping device in a scalable manner, could be applied to quantum information processing[4], simulation[5,10] and spectroscopy[3,11].

better control of field amplitudes and phases, and the absence of spontaneous-emission decoherence[4,16]. Furthermore, microwave near-field control could be incorporated in a chip-level library of transport, junction, storage and microwave manipulation components[22] to advance the integration of quantum control in scalable quantum information processing or simulation.

Our apparatus comprises a room-temperature (300 K) surface-electrode ion trap[23] with 10-μm-thick gold electrodes, separated by gaps of 4.5 μm, deposited onto an insulating AlN substrate (Fig. 1). An oscillating potential (amplitude, 25–50 V; frequency, $f_{RF} = 70.97$ MHz), applied to the radio-frequency electrodes (Fig. 1, RF1 and RF2), provides pseudopotential confinement of $^{25}Mg^{+}$ ions in the radial (x and z) directions at a distance of 30 μm from the electrode surface (all other electrodes are held at radio-frequency ground). Along the trap y axis, ions are confined with static potentials applied to control electrodes C1, C3, C4 and C6. The radial field resulting from these potentials is compensated for by suitable potentials applied to electrodes C2 and C5. Single-ion oscillation frequencies along the y direction can be adjusted between 300 kHz and 2 MHz, and radial frequencies can be adjusted between 4 and 10 MHz. Microwave electrodes MW1, MW2 and MW3 support currents, of order 100 mA to 1 A, that produce an oscillating magnetic field, B_{osc}, above the surface and are used to implement quantum control. To minimize thermal effects during these microwave

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Quantum gates and memory using microwave-dressed states

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Trapped atomic ions have been used successfully to demonstrate[1] basic elements of universal quantum information processing. Nevertheless, scaling up such methods to achieve large-scale, universal quantum information processing (or more specialized quantum simulations[2–5]) remains challenging. The use of easily controllable and stable microwave sources, rather than complex laser systems[6,7], could remove obstacles to scalability. However, the microwave approach has drawbacks: it involves the use of magnetic-field-sensitive states, which shorten coherence times considerably, and requires large, stable magnetic field gradients. Here we show how to overcome both problems by using stationary atomic quantum states as qubits that are induced by microwave fields (that is, by dressing magnetic-field-sensitive states with microwave fields). This permits fast quantum logic, even in the presence of a small (effective) Lamb–Dicke parameter (and, therefore, moderate magnetic field gradients). We experimentally demonstrate the basic building blocks of this scheme, showing that the dressed states are long lived and that coherence times are increased by more than two orders of magnitude relative to those of bare magnetic-field-sensitive states. This improves the prospects of microwave-driven ion trap quantum information processing, and offers a route to extending coherence times in all systems that suffer from magnetic noise, such as neutral atoms, nitrogen-vacancy centres, quantum dots or circuit quantum electrodynamic systems.

forming a decoherence-free subspace have been created[17,18]. Recently, transfer between field-sensitive states used for conditional quantum dynamics and field-insensitive states used for storage of quantum information has been utilized[19].

The relevant noise source in this case, namely magnetic field fluctuations, is not featureless white noise but tends to have a limited bandwidth. In this context, techniques were proposed for prolonging coherence times by subjecting the system to a rapid succession of pulses leading to a decoupling from the environment. This technique, termed bang–bang control[20], and its continuous version[21] can be applied usefully in a variety of systems including hybrid atomic physics and nanophysics technologies. Recent work includes the experimental demonstration of optimized pulse sequences made for suppression of qubit decoherence (see refs 22, 23 and references therein).

Here we encode qubits in microwave-dressed states requiring only continuous, constant-intensity microwave fields. This scheme protects qubits from magnetic field fluctuations and, importantly, simultaneously allows fast quantum logic even for small Lamb–Dicke parameters and, therefore, moderate magnetic field gradients. Microwave-generating elements for the coherent manipulation of qubits can be integrated into microstructured ion traps[7] such that QIP can be realized using scalable ion chips. Thus, this novel scheme is a significant step towards the integration of elements required for QIP on a scalable ion chip. Moreover, the ideas presented here are generic and can be applied to

Foundation of Statistical Mechanics under Experimentally Realistic Conditions

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We demonstrate the equilibration of isolated macroscopic quantum systems, prepared in nonequilibrium mixed states with a significant population of many energy levels, and observed by instruments with a reasonably bound working range compared to the resolution limit. Both properties are satisfied under many, if not all, experimentally realistic conditions. At equilibrium, the predictions and limitations of statistical mechanics are recovered.

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Quantum mechanical evolution towards thermal equilibrium

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The circumstances under which a system reaches thermal equilibrium, and how to derive this from basic dynamical laws, has been a major question from the very beginning of thermodynamics and statistical mechanics. Despite considerable progress, it remains an open problem. Motivated by this issue, we address the more general question of equilibration. We prove, with virtually full generality, that reaching equilibrium is a universal property of quantum systems: almost any subsystem in interaction with a large enough bath will reach an equilibrium state and remain close to it for almost all times. We also prove several general results about other aspects of thermalization besides equilibration, for example, that the equilibrium state does not depend on the detailed microstate of the bath.

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